

Relations and Functions

Question 1.

The function $f: A \rightarrow B$ defined by $f(x) = 4x + 7, x \in \mathbb{R}$ is

- (a) one-one
- (b) Many-one
- (c) Odd
- (d) Even

Answer:

- (a) one-one

Question 2.

The smallest integer function $f(x) = [x]$ is

- (a) One-one
- (b) Many-one
- (c) Both (a) & (b)
- (d) None of these

Answer:

- (b) Many-one

Question 3.

The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 - 4x$ is

- (a) Onto
- (b) Not onto
- (c) None one-one
- (d) None of these

Answer:

- (a) Onto

Question 4.

The number of bijective functions from set A to itself when A contains 106 elements is

- (a) 106
- (b) $(106)^2$
- (c) $106!$
- (d) 2^{106}

Answer:

(c) 106!

Question 5.

If $f(x) = (ax^2 + b)^3$, then the function g such that $f(g(x)) = g(f(x))$ is given by

(a) $g(x) = \left(\frac{b-x^{1/3}}{a}\right)$

(b) $g(x) = \frac{1}{(ax^2+b)^3}$

(c) $g(x) = (ax^2 + b)^{1/3}$

(d) $g(x) = \left(\frac{x^{1/3}-b}{a}\right)^{1/2}$

Answer:

(d) $g(x) = \left(\frac{x^{1/3}-b}{a}\right)^{1/2}$

Question 6.

If $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ and $h : \mathbb{R} \rightarrow \mathbb{R}$ is such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $[h \circ (g \circ f)](x)$, if $x = \frac{\sqrt{\pi}}{2}$ will be

(a) 0

(b) 1

(c) -1

(d) 10

Answer:

(a) 0

Question 7.

If $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the value of x for which $f(g(x)) = 25$ is

(a) ± 1

(b) ± 2

(c) ± 3

(d) ± 4

Answer:

(b) ± 2

Question 8.

Let $f : \mathbb{N} \rightarrow \mathbb{R} : f(x) = \frac{(2x-1)}{2}$ and $g : \mathbb{Q} \rightarrow \mathbb{R} : g(x) = x + 2$ be two functions. Then, $(g \circ f) \left(\frac{3}{2}\right)$ is

(a) 3

(b) 1

(c) $\frac{7}{2}$

(d) None of these

Answer:

(a) 3

Question 9.

Let $f(x) = \frac{x-1}{x+1}$, then $f(f(x))$ is

(a) $\frac{1}{x}$

(b) $-\frac{1}{x}$

(c) $\frac{1}{x+1}$

(d) $\frac{1}{x-1}$

Answer:

(b) $-\frac{1}{x}$

Question 10.

If $f(x) = 1 - \frac{1}{x}$, then $f(f(\frac{1}{x}))$

(a) $\frac{1}{x}$

(b) $\frac{1}{1+x}$

(c) $\frac{x}{x-1}$

(d) $\frac{1}{x-1}$

Answer:

(c) $\frac{x}{x-1}$

Question 11.

If $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ and $h : \mathbb{R} \rightarrow \mathbb{R}$ are such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $(g \circ (f \circ h))(x)$, if $x = 1$ will be

(a) 0

(b) 1

(c) -1

(d) π

Answer:

(a) 0

Question 12.

If $f(x) = \frac{3x+2}{5x-3}$ then $(f \circ f)(x)$ is

(a) x

(b) -x

(c) f(x)

(d) -f(x)

Answer:

(a) x

Question 13.

If the binary operation $*$ is defined on the set Q^+ of all positive rational numbers by $a * b = \frac{ab}{4}$.

Then, $3 * \left(\frac{1}{5} * \frac{1}{2}\right)$ is equal to

- (a) $\frac{3}{160}$
- (b) $\frac{5}{160}$
- (c) $\frac{3}{10}$
- (d) $\frac{3}{40}$

Answer:

- (a) $\frac{3}{160}$

Question 14.

The number of binary operations that can be defined on a set of 2 elements is

- (a) 8
- (b) 4
- (c) 16
- (d) 64

Answer:

- (c) 16

Question 15.

Let $*$ be a binary operation on Q , defined by $a * b = \frac{3ab}{5}$ is

- (a) Commutative
- (b) Associative
- (c) Both (a) and (b)
- (d) None of these

Answer:

- (c) Both (a) and (b)

Question 16.

Let $*$ be a binary operation on set Q of rational numbers defined as $a * b = \frac{ab}{5}$. Write the identity for $*$.

- (a) 5
- (b) 3
- (c) 1
- (d) 6

Answer:

- (a) 5

Question 17.

For binary operation $*$ defined on $R - \{1\}$ such that $a * b = \frac{a}{b+1}$ is

- (a) not associative
- (b) not commutative
- (c) commutative
- (d) both (a) and (b)

Answer:

- (d) both (a) and (b)

Question 18.

The binary operation $*$ defined on set R , given by $a * b = \frac{a+b}{2}$ for all $a, b \in R$ is

- (a) commutative
- (b) associative
- (c) Both (a) and (b)
- (d) None of these

Answer:

- (a) commutative

Question 19.

Let $A = N \times N$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Then $*$ is

- (a) commutative
- (b) associative
- (c) Both (a) and (b)
- (d) None of these

Answer:

- (c) Both (a) and (b)

Question 20.

Find the identity element in the set I^+ of all positive integers defined by $a * b = a + b$ for all $a, b \in I^+$.

- (a) 1
- (b) 2
- (c) 3
- (d) 0

Answer:

- (d) 0

Question 21.

Let $*$ be a binary operation on set $Q - \{1\}$ defined by $a * b = a + b - ab$: $a, b \in Q - \{1\}$. Then $*$ is

- (a) Commutative
- (b) Associative
- (c) Both (a) and (b)
- (d) None of these

Answer:

(c) Both (a) and (b)

Question 22.

The binary operation $*$ defined on N by $a * b = a + b + ab$ for all $a, b \in N$ is

- (a) commutative only
- (b) associative only
- (c) both commutative and associative
- (d) none of these

Answer:

(c) both commutative and associative

Question 23.

The number of commutative binary operation that can be defined on a set of 2 elements is

- (a) 8
- (b) 6
- (c) 4
- (d) 2

Answer:

(d) 2

Question 24.

Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to $b \forall a, b \in T$. Then R is

- (a) reflexive but not transitive
- (b) transitive but not symmetric
- (c) equivalence
- (d) None of these

Answer:

(c) equivalence

Question 25.

The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are

- (a) 1
- (b) 2
- (c) 3
- (d) 5

Answer:

(d) 5

Question 26.

Let us define a relation R in R as aRb if $a \geq b$. Then R is

- (a) an equivalence relation

- (b) reflexive, transitive but not symmetric
- (c) symmetric, transitive but not reflexive
- (d) neither transitive nor reflexive but symmetric

Answer:

- (b) reflexive, transitive but not symmetric

Question 27.

Let $A = \{1, 2, 3\}$ and consider the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$. Then R is

- (a) reflexive but not symmetric
- (b) reflexive but not transitive
- (c) symmetric and transitive
- (d) neither symmetric, nor transitive

Answer:

- (a) reflexive but not symmetric

Question 28.

The identity element for the binary operation $*$ defined on $Q - \{0\}$ as $a * b = \frac{ab}{2} \forall a, b \in Q - \{0\}$ is

- (a) 1
- (b) 0
- (c) 2
- (d) None of these

Answer:

- (c) 2

Question 29.

Let $A = \{1, 2, 3, \dots, n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is

- (a) ${}^n P_2$
- (b) $2^n - 2$
- (c) $2^n - 1$
- (d) none of these

Answer:

- (b) $2^n - 2$

Question 30.

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{1}{x} \forall x \in \mathbb{R}$. Then f is

- (a) one-one
- (b) onto
- (c) bijective
- (d) f is not defined

Answer:

(d) f is not defined

Question 31.

Which of the following functions from Z into Z are bijective?

(a) $f(x) = x^3$

(b) $f(x) = x + 2$

(c) $f(x) = 2x + 1$

(d) $f(x) = x^2 + 1$

Answer:

(b) $f(x) = x + 2$

Question 32.

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the functions defined by $f(x) = x^3 + 5$. Then $f^{-1}(x)$ is

(a) $(x + 5)^{\frac{1}{3}}$

(b) $(x - 5)^{\frac{1}{3}}$

(c) $(5 - x)^{\frac{1}{3}}$

(d) $5 - x$

Answer:

(b) $(x - 5)^{\frac{1}{3}}$

Question 33.

Let $f : \mathbb{R} - \{\frac{3}{5}\} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{3x+2}{5x-3}$. Then

(a) $f^{-1}(x) = f(x)$

(b) $f^{-1}(x) = -f(x)$

(c) $(f \circ f) x = -x$

(d) $f^{-1}(x) = \frac{1}{19} f(x)$

Answer:

(a) $f^{-1}(x) = f(x)$

Question 34.

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \tan x$. Then $f^{-1}(1)$ is

(a) $\frac{\pi}{4}$

(b) $\{n\pi + \frac{\pi}{4}; n \in \mathbb{Z}\}$

(c) Does not exist

(d) None of these

Answer:

(b) $\{n\pi + \frac{\pi}{4}; n \in \mathbb{Z}\}$

Question 35.

Let R be a relation on the set N of natural numbers denoted by $nRm \Leftrightarrow n$ is a factor of m (i.e. $n \mid m$). Then, R is

- (a) Reflexive and symmetric
- (b) Transitive and symmetric
- (c) Equivalence
- (d) Reflexive, transitive but not symmetric

Answer:

- (d) Reflexive, transitive but not symmetric

Question 36.

Let $S = \{1, 2, 3, 4, 5\}$ and let $A = S \times S$. Define the relation R on A as follows:

(a, b) R (c, d) iff $ad = cb$. Then, R is

- (a) reflexive only
- (b) Symmetric only
- (c) Transitive only
- (d) Equivalence relation

Answer:

- (d) Equivalence relation

Question 37.

Let R be the relation “is congruent to” on the set of all triangles in a plane is

- (a) reflexive
- (b) symmetric
- (c) symmetric and reflexive
- (d) equivalence

Answer:

- (d) equivalence

Question 38.

Total number of equivalence relations defined in the set $S = \{a, b, c\}$ is

- (a) 5
- (b) $3!$
- (c) 23
- (d) 33

Answer:

- (a) 5

Question 39.

The relation R is defined on the set of natural numbers as $\{(a, b) : a = 2b\}$. Then, R^{-1} is given by

- (a) $\{(2, 1), (4, 2), (6, 3), \dots\}$
- (b) $\{(1, 2), (2, 4), (3, 6), \dots\}$
- (c) R^{-1} is not defined

(d) None of these

Answer:

(b) $\{(1, 2), (2, 4), (3, 6), \dots\}$

Question 40.

Let $X = \{-1, 0, 1\}$, $Y = \{0, 2\}$ and a function $f: X \rightarrow Y$ defined by $y = 2x^4$, is

(a) one-one onto

(b) one-one into

(c) many-one onto

(d) many-one into

Answer:

(c) many-one onto

Question 41.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$ then $f(x)$ is

(a) one-one onto

(b) one-one but not onto

(c) onto but not one-one

(d) None of these

Answer:

(d) None of these

Question 42.

Let $g(x) = x^2 - 4x - 5$, then

(a) g is one-one on $\mathbb{R}$

(b) g is not one-one on $\mathbb{R}$

(c) g is bijective on $\mathbb{R}$

(d) None of these

Answer:

(b) g is not one-one on $\mathbb{R}$

Question 43.

Let $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$. Let $f: A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$. Then,

(a) f is bijective

(b) f is one-one but not onto

(c) f is onto but not one-one

(d) None of these

Answer:

(a) f is bijective

Question 44.

The mapping $f: \mathbb{N} \rightarrow \mathbb{N}$ is given by $f(n) = 1 + n^2$, $n \in \mathbb{N}$ when $\mathbb{N}$ is the set of natural numbers is

- (a) one-one and onto
- (b) onto but not one-one
- (c) one-one but not onto
- (d) neither one-one nor onto

Answer:

- (c) one-one but not onto

Question 45.

The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3 - 1$ is

- (a) a one-one function
- (b) an onto function
- (c) a bijection
- (d) neither one-one nor onto

Answer:

- (c) a bijection

Question 46.

Let $f: [0, \infty) \rightarrow [0, 2]$ be defined by $f(x) = \frac{2x}{1+x}$, then f is

- (a) one-one but not onto
- (b) onto but not one-one
- (c) both one-one and onto
- (d) neither one-one nor onto

Answer:

- (a) one-one but not onto

Question 47.

If $\mathbb{N}$ be the set of all-natural numbers, consider $f: \mathbb{N} \rightarrow \mathbb{N}$ such that $f(x) = 2x, \forall x \in \mathbb{N}$, then f is

- (a) one-one onto
- (b) one-one into
- (c) many-one onto
- (d) None of these

Answer:

- (b) one-one into

Question 48.

Let $A = \{x: -1 \leq x \leq 1\}$ and $f: A \rightarrow A$ is a function defined by $f(x) = x|x|$ then f is

- (a) a bijection
- (b) injection but not surjection
- (c) surjection but not injection
- (d) neither injection nor surjection

Answer:

- (a) a bijection

Question 49.

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = x^3 + 4$, then f is

- (a) injective
- (b) surjective
- (c) bijective
- (d) none of these

Answer:

- (c) bijective

Question 50.

If $f(x) = (ax^2 - b)^3$, then the function g such that $f\{g(x)\} = g\{f(x)\}$ is given by

- (a) $g(x) = \left(\frac{b-x^{1/3}}{a}\right)^{1/2}$
- (b) $g(x) = \frac{1}{(ax^2+b)^3}$
- (c) $g(x) = (ax^2 + b)^{1/3}$
- (d) $g(x) = \left(\frac{x^{1/3}+b}{a}\right)^{1/2}$

Answer:

- (d) $g(x) = \left(\frac{x^{1/3}+b}{a}\right)^{1/2}$

Question 51.

If $f : [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then f^{-1} equals to

- (a) $\frac{x+\sqrt{x^2-4}}{2}$
- (b) $\frac{x}{1+x^2}$
- (c) $\frac{x-\sqrt{x^2-4}}{2}$
- (d) $1 + \sqrt{x^2 - 4}$

Answer:

- (a) $\frac{x+\sqrt{x^2-4}}{2}$

Question 52.

Let $f(x) = x^2 - x + 1$, $x \geq \frac{1}{2}$, then the solution of the equation $f(x) = f^{-1}(x)$ is

- (a) $x = 1$
- (b) $x = 2$
- (c) $x = \frac{1}{2}$
- (d) None of these

Answer:

- (a) $x = 1$

Question 53.

Which one of the following function is not invertible?

(a) $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x + 1$

(b) $f: \mathbb{R} \rightarrow [0, \infty), f(x) = x^2$

(c) $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+, f(x) = \frac{1}{x^3}$

(d) None of these

Answer:

(d) None of these

Question 54.

The inverse of the function $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is

(a) $\log_{10}(2 - x)$

(b) $\frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$

(c) $\frac{1}{2} \log_{10}(2x - 1)$

(d) $\frac{1}{4} \log \left(\frac{2x}{2-x} \right)$

Answer:

(b) $\frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$

Question 55.

If $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x-7}{4}$ is an invertible function, then find f^{-1} .

(a) $\frac{4x+5}{2}$

(b) $\frac{4x+7}{2}$

(c) $\frac{3x+2}{2}$

(d) $\frac{9x+3}{5}$

Answer:

(b) $\frac{4x+7}{2}$

Question 56.

Consider the function f in $A = \mathbb{R} - \left\{ \frac{2}{3} \right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$. Find f^{-1} .

(a) $\frac{3+4x}{6x-4}$

(b) $\frac{6x-4}{3+4x}$

(c) $\frac{3-4x}{6x-4}$

(d) $\frac{9+2x}{6x-4}$

Answer:

(a) $\frac{3+4x}{6x-4}$

Question 57.

If f is an invertible function defined as $f(x) = \frac{3x-4}{5}$, then $f^{-1}(x)$ is

- (a) $5x + 3$
- (b) $5x + 4$
- (c) $\frac{5x+4}{3}$
- (d) $\frac{3x+2}{3}$

Answer:

- (c) $\frac{5x+4}{3}$

Question 58.

If $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{3x+5}{2}$ is an invertible function, then find f^{-1} .

- (a) $\frac{2x-5}{3}$
- (b) $\frac{x-5}{3}$
- (c) $\frac{5x-2}{3}$
- (d) $\frac{x-2}{3}$

Answer:

- (a) $\frac{2x-5}{3}$

Question 59.

Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ be two functions such that $f(x) = 2x - 3$, $g(x) = x^3 + 5$. The function $(f \circ g)^{-1}(x)$ is equal to

- (a) $\left(\frac{x+7}{2}\right)^{1/3}$
- (b) $\left(x - \frac{7}{2}\right)^{1/3}$
- (c) $\left(\frac{x-2}{7}\right)^{1/3}$
- (d) $\left(\frac{x-7}{2}\right)^{1/3}$

Answer:

- (d) $\left(\frac{x-7}{2}\right)^{1/3}$

Question 60.

Let $*$ be a binary operation on set of integers I , defined by $a * b = a + b - 3$, then find the value of $3 * 4$.

- (a) 2
- (b) 4
- (c) 7

(d) 6

Answer:

(c) 7

Question 61.

If $*$ is a binary operation on set of integers I defined by $a * b = 3a + 4b - 2$, then find the value of $4 * 5$.

(a) 35

(b) 30

(c) 25

(d) 29

Answer:

(b) 30

Question 62.

Let $*$ be the binary operation on N given by $a * b = \text{HCF}(a, b)$ where, $a, b \in N$. Find the value of $22 * 4$.

(a) 1

(b) 2

(c) 3

(d) 4

Answer:

(b) 2

Question 63.

Consider the binary operation $*$ on Q defined by $a * b = a + 12b + ab$ for $a, b \in Q$. Find $2 * \frac{1}{3}$

(a) $\frac{20}{3}$

(b) 4

(c) 18

(d) $\frac{16}{3}$

Answer:

(a) $\frac{20}{3}$

Question 64.

The domain of the function $f(x) = \frac{1}{\sqrt{\{\sin x\} + \{\sin(\pi+x)\}}}$ where $\{.\}$ denotes fractional part, is

(a) $[0, \pi]$

(b) $(2n+1)\pi/2, n \in Z$

(c) $(0, \pi)$

(d) None of these

Answer:

(d) None of these

Question 65.

Range of $f(x) = \sqrt{(1 - \cos x) \sqrt{(1 - \cos x) \sqrt{(1 - \cos x) \dots \infty}}}$

- (a) $[0, 1]$
- (b) $(0, 1)$
- (c) $[0, 2]$
- (d) $(0, 2)$

Answer:

- (c) $[0, 2]$